



Maple Conference

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# Using GeoGebra Discovery and ChatGPT for dealing with geometric statements

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# Our aim

The GeoGebra logo, featuring the word "GeoGebra" in a grey sans-serif font. The "o" in "Geo" is replaced by a geometric diagram consisting of a circle with several points on its circumference, some of which are connected by lines.

Automated tools for Geometry



# ChatGPT

LLM – Natural language processing

## Confront

Analyze and compare the behavior of ChatGPT and GeoGebra Discovery dealing with geometric reasoning



## Collaborate

Propose a cooperation as complementary systems to analyze and to develop in a near future

# In this talk

GeoGebra



- Some short History
- Solving Gelernter's problems with the help of Maple
- Discussion
- Conclusions

# Short History: The 1950's origins

Since the birth of Artificial Intelligence (AI), mathematics have special attention as a field for testing related systems and programs

[1957] **Logic machines** for math reasoning program: **The Logic Theorist** proves 38 out of the 52 theorems in Chapter 2 of the Russell and Whitehead's Principia Mathematica, returning shorter proofs for some theorems.

[1959] **Automatic geometry reasoning**: H. **Gelernter** presented at a UNESCO meeting in Paris a first geometry-theorem proving machine.

# Short History: Gelernter's forecasts

“In early spring, 1959, an IBM 704 computer, with the assistance of a program comprising some 20,000 individual instructions, proved its first theorem in elementary Euclidean plane geometry . Since that time, the geometry theorem-proving machine (a particular state configuration of the IBM 704 specified by the afore-mentioned machine code) has found solutions to a large number of problems taken from high school textbooks and final examinations in plane geometry . . . the theorem-proving program relies upon heuristic methods to restrain it from generating proof sequences that do not have a high a priori probability of leading to a proof for the theorem in question . . . At this stage in its development, the geometry machine was capable of producing proofs that were quite impressive (Appendix I).. . . Three years ago, the dominant opinion was that the geometry machine would not exist today. And today, hardly an expert will contest the assertion that machines will be proving interesting theorems in number theory three years hence.. . .”

H. Gelernter (1960)

# But... (not three!) sixty years later

## **2020's Explosion of AI tools for dealing with mathematical issues**

Some examples:

- [2021] Lean Theorem Prover: integrating automated tools and methods, supporting user interaction and the construction of fully specified axiomatic proofs.
- [2024] Solving mathematics olympiad problems with AlphaGeometry, a neural language model.
- [2024] Prepare lesson plans in primary school mathematic courses with ChatGPT.

# ChatGPT & Geogebra Discovery

ChatGPT	GeoGebra Discovery
<ul style="list-style-type: none"><li>• LLMs has facilitated their dissemination and exploration in different contexts.</li><li>• ChatGPT (released 2022) has been rapidly tested in several fields of knowledge, thanks to its suitability and accuracy</li><li>• Regarding mathematics, the main point of attention in maths has been on teaching and learning issues</li></ul>	<ul style="list-style-type: none"><li>• Fork version of GeoGebra.</li><li>• It is available online for different OS and an on-line version.</li><li>• “Symbolic geometry calculator” to conjecture, discover and prove statements based on complex and real algebraic geometry</li><li>• Recently, including proof certificates</li><li>• Under development, not yet included in the official GeoGebra version</li></ul>

GeoGebra Discovery's website

<https://github.com/kovzol/geogebra-discovery#geogebra-discovery>

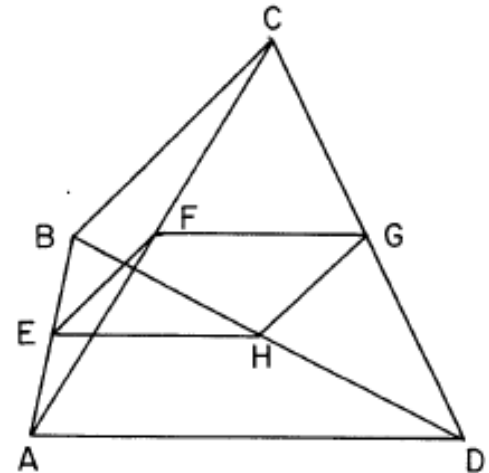
# Gelernter's Theorem 1

```
PROOF
*****
SEGMENT DG EQUALS SEGMENT GC
  DEFINITION OF MIDPOINT
SEGMENT CF EQUALS SEGMENT FA
  DEFINITION OF MIDPOINT
TRIANGLE DCA
  ASSUMPTION BASED ON DIAGRAM
PRECEDES DGC
  DEFINITION OF MIDPOINT
PRECEDES CFA
  DEFINITION OF MIDPOINT
SEGMENT GF PARALLEL SEGMENT AD
  SEGMENT JOINING MIDPOINTS OF SIDES OF TRIANGLE IS PARALLEL TO BASE
SEGMENT HE PARALLEL SEGMENT AD
  SYNTACTIC CONJUGATE
SEGMENT GF PARALLEL SEGMENT EH
  SEGMENTS PARALLEL TO THE SAME SEGMENT ARE PARALLEL
SEGMENT HG PARALLEL SEGMENT FE
  SYNTACTIC CONJUGATE
QUAD-LATERAL HGFE
  ASSUMPTION BASED ON DIAGRAM
PARALELOGRAM EFGH
  QUADRILATERAL WITH OPPOSITE SIDES PARALLEL IS A PARALLELOGRAM
```

```
PREMISES
*****
QUAD-LATERAL ABCD
POINT E MIDPOINT SEGMENT AB
POINT F MIDPOINT SEGMENT AC
POINT G MIDPOINT SEGMENT CD
POINT H MIDPOINT SEGMENT BD
```

```
TO PROVE
*****
PARALELOGRAM EFGH
```

```
SYNTACTIC SYMMETRIES
*****
BA, AB, DC, CD, EE, HF, GG, FH,
CA, DB, AC, BD, GE, FF, EG, HH,
DA, CB, BC, AD, GE, HF, EG, FH,
```



TOTAL ELAPSED TIME 1.03 MINUTES



# Gelernter's Thm. 1 with ChatGPT



Since  $E$  and  $F$  are midpoints of sides  $AB$  and  $AC$  respectively, by the midpoint theorem,  $EF$  is parallel to  $BC$ .

Similarly, since  $G$  and  $H$  are midpoints of sides  $CD$  and  $BD$  respectively,  $GH$  is parallel to  $BC$  by the midpoint theorem.

Since  $EF$  is parallel to  $BC$  and  $GH$  is also parallel to  $BC$ , by transitivity of parallel lines,  $EF$  is parallel to  $GH$ .

Now, let's consider  $EH$  and  $FG$ .  $EH$  is a midsegment of triangle  $ACD$  and  $FG$  is a midsegment of triangle  $ABC$ .

By the midpoint theorem,  $EH$  is parallel to  $AD$  and  $FG$  is parallel to  $AB$ .

Since  $AD$  is parallel to  $BC$  (as opposite sides of quadrilateral  $ABCD$ ),  $EH$  is parallel to  $BC$ .  
Thus, we have  $EF \parallel GH$  and  $EH \parallel FG$ .  
Since opposite sides are parallel,  $EFGH$  is a parallelogram.

1. Consider triangle  $ABD$ :

$E$  and  $H$  are the midpoints of  $AB$  and  $BD$  respectively. By the midpoint theorem, line segment  $EH$  is parallel to  $AD$  and  $EH = \frac{1}{2}AD$ .

2. Consider triangle  $ACD$ :

$F$  and  $G$  are the midpoints of  $AC$  and  $CD$  respectively. By the midpoint theorem, segment  $FG$  is parallel to  $AD$  and  $FG = \frac{1}{2}AD$ .

Since  $EH$  and  $FG$  are both parallel to  $AD$  and have the same length ( $\frac{1}{2}AD$ ), that  $EH \parallel FG$  and  $EH = FG$ .

3. Consider triangle  $ABC$ :

$E$  and  $F$  are the midpoints of  $AB$  and  $AC$  respectively. By the midpoint theorem, segment  $EF$  is parallel to  $BC$  and  $EF = \frac{1}{2}BC$ .

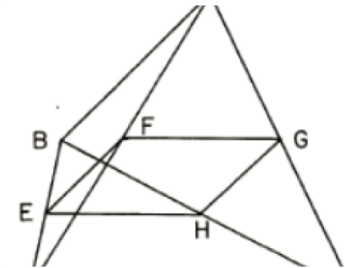
4. Consider triangle  $BCD$ :

$H$  and  $G$  are the midpoints of  $BD$  and  $CD$  respectively. By the midpoint theorem, segment  $HG$  is parallel to  $BC$  and  $HG = \frac{1}{2}BC$ .

Since  $EF$  and  $HG$  are both parallel to  $BC$  and have the same length ( $\frac{1}{2}BC$ ), that  $EF \parallel HG$  and  $EF = HG$ .

Therefore, we have shown that  $EH \parallel FG$  and  $EF \parallel HG$  with  $EH = FG$  and  $EF = HG$ . This means that  $EFGH$  has both pairs of opposite sides parallel and equal in length, which is the definition of a parallelogram.

Hence,  $EFGH$  is a parallelogram.



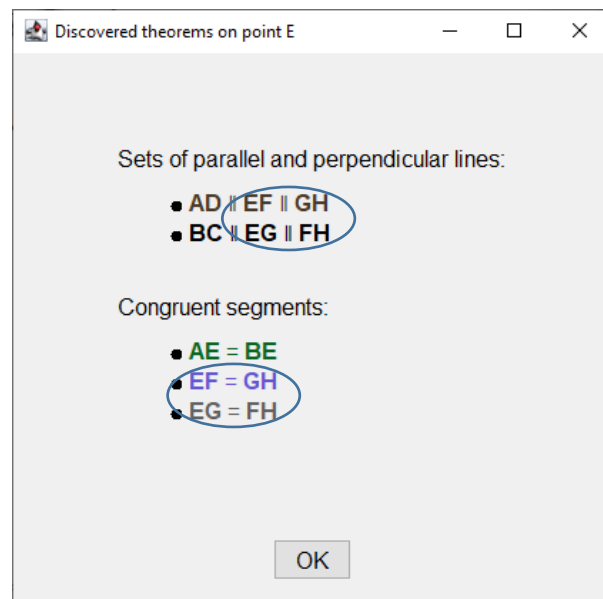
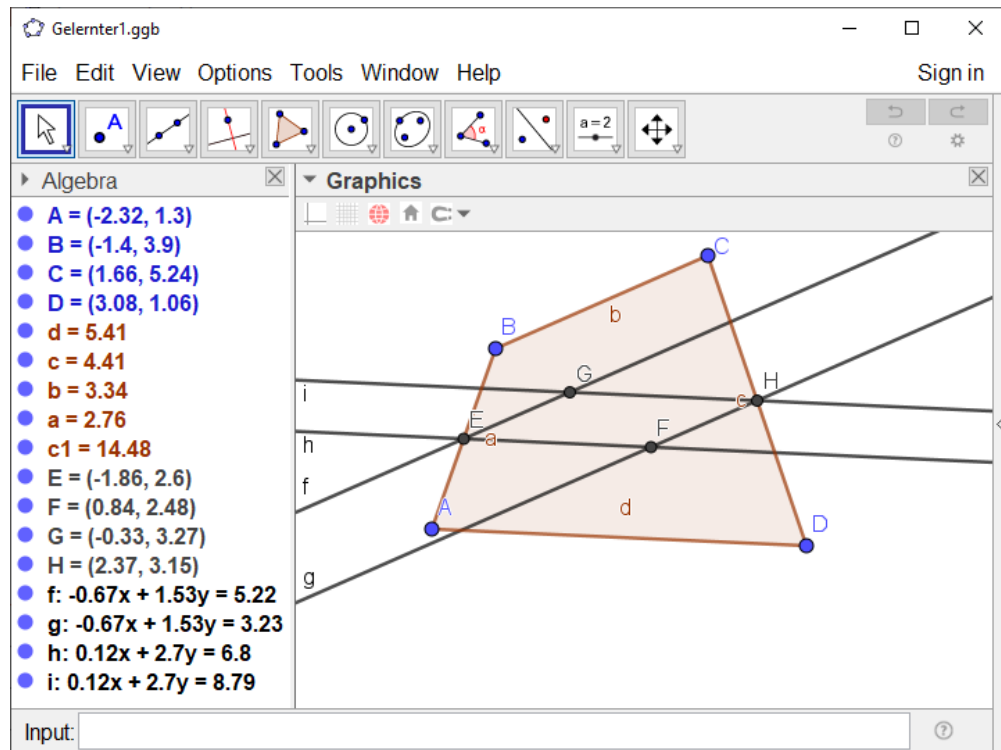
Consider the following figure. Prove that  $EFGH$  is a parallelogram.

To prove that  $EFGH$  is a parallelogram in the given figure, we'll use the properties of midpoints and vectors as previously outlined. Let's identify and label the points clearly from the image:

- $A, B, C$ , and  $D$  are the vertices of the quadrilateral.
- $E$  is the midpoint of  $AB$ .
- $G$  is the midpoint of  $CD$ .
- $F$  is the midpoint of  $BC$ .
- $H$  is the midpoint of  $AD$ .



# Gelernter's Thm. 1 with GGb Discovery





# A proof-certificate: ShowProof(AreParallel(f,g))

CAS

1

Let A, B, C, D be arbitrary points.

2

Let c1 be the polygon A, B, C, D.

3

Let a be the segment A, B.

4

Let c be the segment C, D.

5

Let E be the midpoint of a.

6

Let F be the midpoint of B, D.

7

Let G be the midpoint of C, A.

8

Let H be the midpoint of c.

9

Let f be the line E, G.

10

Let g be the line F, H.

11

Prove that AreParallel(f, g).

12

The statement is true under some non-degeneracy conditions (see

13

We prove this by contradiction.

Graphics



CAS	
14	Let free point A be denoted by $(v1,v2)$ .
15	Let free point B be denoted by $(v3,v4)$ .
16	Let free point C be denoted by $(v5,v6)$ .
17	Let free point D be denoted by $(v7,v8)$ .
18	Considering definition E = Midpoint(a):
19	Let dependent point E be denoted by $(v9,v10)$ .
20	$e1:=2*v9-v3-v1=0$ $\rightarrow e1: -v1 - v3 + 2 v9 = 0$
21	$e2:=2*v10-v4-v2=0$ $\rightarrow e2: 2 v10 - v2 - v4 = 0$
22	Considering definition F = Midpoint(B, D):
23	Let dependent point F be denoted by $(v11,v12)$ .
24	$e3:=2*v11-v7-v3=0$ $\rightarrow e3: 2 v11 - v3 - v7 = 0$

CAS	
38	$(T1*v17-1)=0$ $\rightarrow v17 (-v10 v11 + v10 v15 + v11$
39	$e9:=-1-v17*v15*v14+v17*v16*v13-v17*v13*v$ $\rightarrow e9: -v10 v11 v17 + v10 v15 v1$
40	Without loss of generality, some coordinates
41	$\{v1=0,v2=0,v3=0,v4=1\}$ $\rightarrow \{v1 = 0, v2 = 0, v3 = 0, v4 = 1\}$
42	The statement requires some conditions:
43	• A and B are not equal
44	All hypotheses and the negated thesis after s
45	$s1:2*v9=0$ $\rightarrow s1: 2 v9 = 0$
46	$s2:-1+2*v10=0$ $\rightarrow s2: 2 v10 - 1 = 0$
<	

CAS	
50	$s6:2*v14-v6=0$ $\rightarrow s6: 2 v14 - v6 = 0$
51	$s7:2*v15-v7-v5=0$ $\rightarrow s7: 2 v15 - v5 - v7 = 0$
52	$s8:2*v16-v8-v6=0$ $\rightarrow s8: 2 v16 - v6 - v8 = 0$
53	$s9:-1-v17*v15*v14+v17*v16*v13-v17*v13*v12+v17*v14*v$ $\rightarrow s9: -v10 v11 v17 + v10 v15 v17 + v11 v1$
54	Now we consider the following equation:
55	$s1*(-1/4*v17*v6+1/4*v17)+s2*(1/4*v17*v5)+s3*(-1/2*v10*v$ $\rightarrow 1 = 0$
56	<b>Contradiction! This proves the original statement.</b>
57	The statement has a difficulty of degree 2.

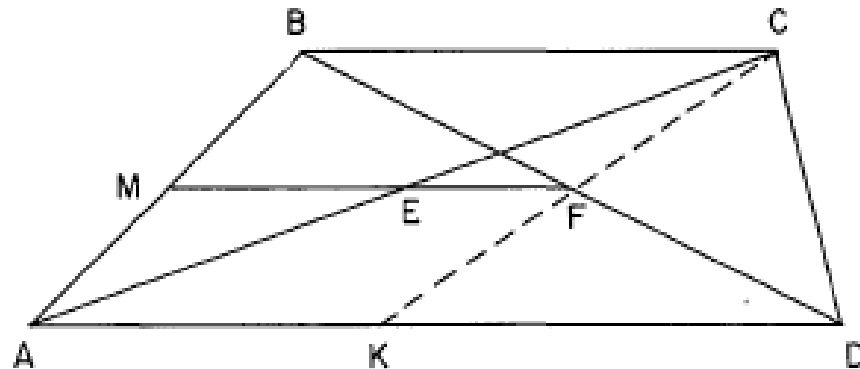
# Gelernter's Thm. 2

## Appendix II

PREMISES  
\*\*\*\*\*  
QUAD-LATERAL ABCD  
SEGMENT BC PARALLEL SEGMENT AD  
POINT E MIDPOINT SEGMENT AC  
POINT F MIDPOINT SEGMENT BD  
PRECEDES MEF  
PRECEDES AMB

TO PROVE  
\*\*\*\*\*  
SEGMENT MB EQUALS SEGMENT MA

NO SYNTACTIC SYMMETRIES  
\*\*\*\*\*



I AM STUCK, ELAPSED TIME 8.12 MINUTES

\*\*\*\*\*  
CONSTRUCT SEGMENT CF  
EXTEND SEGMENT CF TO INTERSECT SEGMENT AD IN POINT K

ADD TO PREMISES THE FOLLOWING STATEMENTS  
\*\*\*\*\*  
PRECEDES CFK  
COLLINEAR AKD

... TOTAL ELAPSED TIME 30.68 MINUTES

# Gelernter's Thm. 2 with ChatGPT



To prove that segment BM equals segment MA in the given configuration of a trapezoid ABCD where  $AD \parallel BC$ , follow these steps:

**Trapezoid Configuration:** Given trapezoid ABCD with  $AD \parallel BC$  and vertices in a clockwise manner. Line f passes through points B and C. Line g passes through point A and is parallel to f, hence  $g \parallel BC$ .

**Midpoints:** E is the midpoint of AC, so  $AE = EC$ . F is the midpoint of BD. Line h passes through points E and F. Line i passes through points M and B.

**Intersection Point:** Let M be the intersection point of lines g and h.

**Properties and Symmetry:** Since  $AD \parallel BC$ , and E and F are midpoints, line h (EF) behaves as a midsegment of trapezoid ABCD.

**Key Property of Midsegment:** The line segment joining the midpoints of the non-parallel sides of a trapezoid (line h) is parallel to the bases (AD and BC) and its length is equal to the sum of the lengths of the bases.

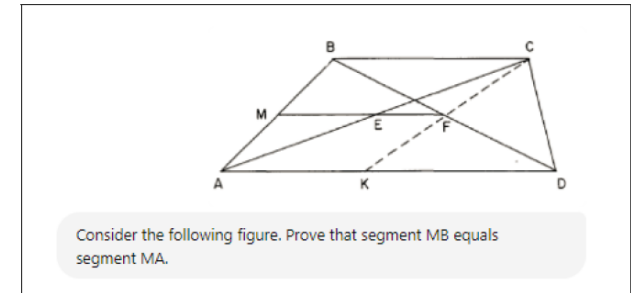
**Geometric Analysis:** Since h is the midsegment, it is parallel to both AD and BC. Line i (MB) is perpendicular to h. This configuration divides the trapezoid into smaller sections that reflect the symmetry of the bases.

**Using the Midpoints E and F:** Midpoint E of AC and midpoint F of BD are used to construct line h, which passes through the center of the trapezoid.

**Intersection and Equal Segments:** Since h connects the midpoints of the diagonals, and line i (MB) is perpendicular to h, M divides AB into two equal segments, BM and MA.

**Detailed Symmetry Argument:** Consider triangles  $\triangle ABE$  and  $\triangle CDE$ . Since E is the midpoint of AC,  $AE = CE$ . Since  $AD \parallel BC$ ,  $\angle BAE = \angle DCE$  and  $\angle ABE = \angle CDE$ . Therefore,  $\triangle ABE \cong \triangle CDE$  by ASA. This implies  $BE = DE$ . Similarly, considering triangles  $\triangle BCF$  and  $\triangle ADF$ , we can show  $BF = DF$ . This symmetry supports the conclusion that M is the midpoint of AB.

**Conclusion:** Since M divides AB symmetrically:  $BM = MA$ .



To prove that segment MB is equal to segment MA in the given trapezoid ABCD with  $AB \parallel CD$ , let's use a combination of coordinate geometry and properties of midpoints in trapezoids. The key insight is to show that M is the midpoint of AB.

Step-by-Step Proof:

1. Identify the Key Points:

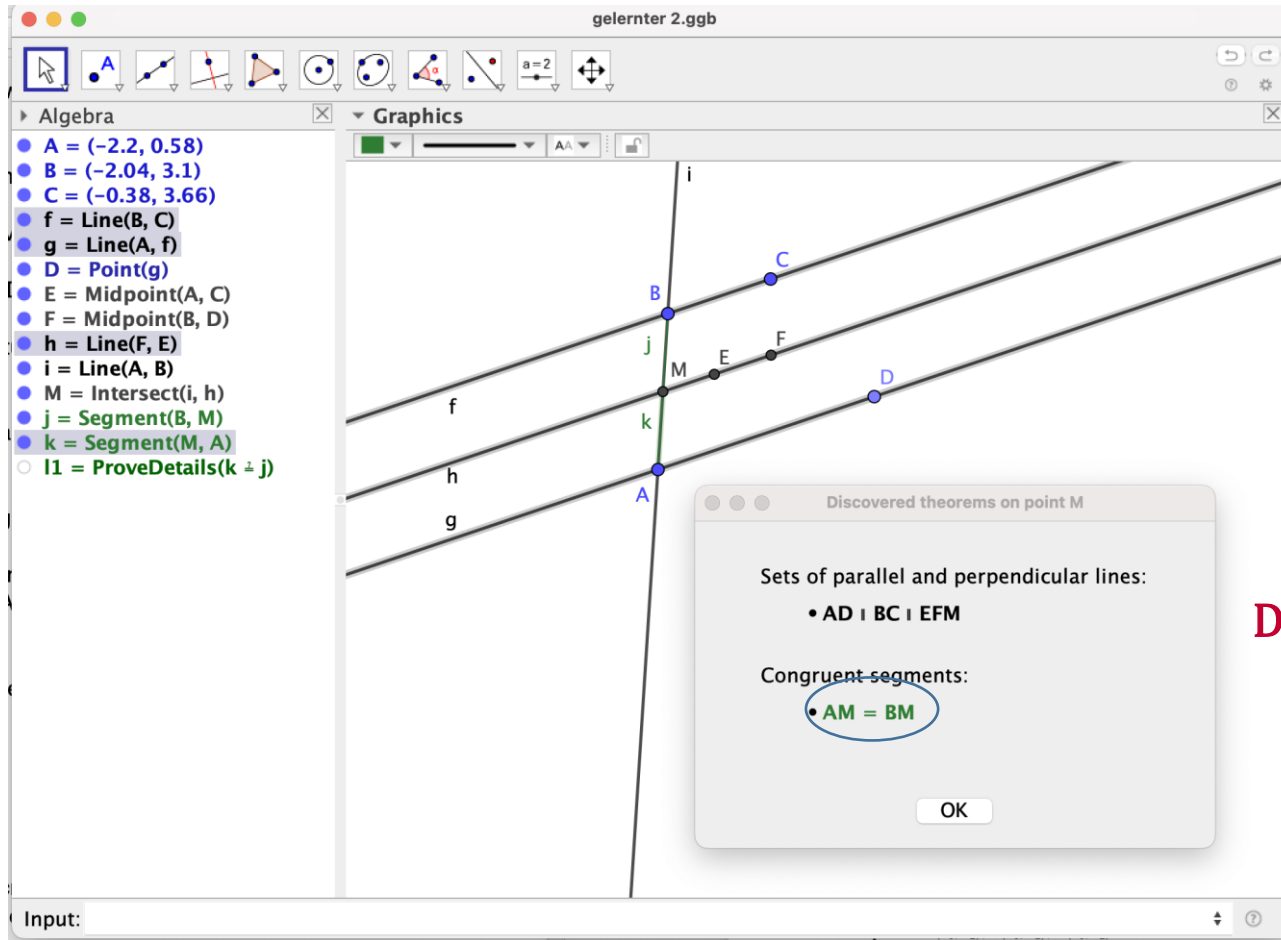
- Let A be at  $(0, 0)$ , B at  $(b, 0)$ , C at  $(c, d)$ , and D at  $(a, d)$ .
- Let E be the midpoint of diagonal AC and F be the midpoint of diagonal BD.
- Let M and K be the midpoints of AB and CD, respectively.

2. Find Coordinates of Midpoints M and K:

- M, the midpoint of AB:

$$M = \left( \frac{0+b}{2}, \frac{0+0}{2} \right) = \left( \frac{b}{2}, 0 \right)$$

# Gelernter's Thm. 2 with GGb Discovery





# I am stuck! $\text{ShowProof}(k=j)$ vs $\text{ProveDetails}(k=j)$

GeoGebra Classic 5

Algebra

- $A = (-2.2, 0.58)$
- $B = (-0.92, 3.08)$
- $C = (3.7, 3.44)$
- $f: -0.36x + 4.62y = 14.56$
- $g: -0.36x + 4.62y = 3.47$
- $D = (5.46, 1.18)$
- $E = (0.75, 2.01)$
- $F = (2.27, 2.13)$
- $h: 0.12x - 1.52y = -2.97$
- $i: -2.5x + 1.28y = 6.24$
- $M = (-1.56, 1.83)$
- $j = 1.4$
- $k = 1.4$
- $l1 = \{\text{true}, \{\dots\}\}$

CAS

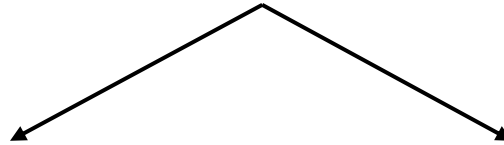
- Let A, B, C be arbitrary points.
- Let f be the line B, C.
- Let g be the line through A parallel to f.
- Let D be a point on g.
- Let E be the midpoint of A, C.
- Let F be the midpoint of B, D.
- Let h be the line F, E.
- Let i be the line A, B.
- Let M be the intersection of i and h.
- Prove that  $k \neq j$ .
- The statement could not be proven nor disproven.
- 
- 

Graphics



# Computer algebra issues

- **True on Parts:** labels statements that are true on some non-degenerate prime components of the hypotheses variety, but are also false on some others.



## Elimination:

$$(H, tT - 1)K[X, t] \cap K[Y] = (0)$$

$$(H, T)K[X] \cap K[Y] = (0)$$

## Ideal primary (prime) decomposition:

$$H = H_1 \cap \cdots \cap H_p$$

Check on each  $H_i$

- **Minimal Extended Polynomial (MEP):** to deal with statements involving variables that are defined through square powers (like the distance between two points or the length of a segment).

# Gelernter's Thm. 2 with Maple: Elimination

```
> EliminationIdeal (<(d1-a1)*(c2-b2)-(d2-a2)*(c1-b1), 2*e1-(a1+c1), 2*e2-(a2+c2), 2*f1-(b1+d1), 2*f2-(b2+d2), (m1-f1)*(f2-e2)-(m2-f2)*(f1-e1), (m1-a1)*(a2-b2)-(m2-a2)*(a1-b1), j^2-(b1-m1)^2-(b2-m2)^2, k^2-(a1-m1)^2-(a2-m2)^2, t*(j-k)-1>, {a1,a2,b1,b2,c1,c2,d1});
```

(0)

(4)

```
> EliminationIdeal (<(d1-a1)*(c2-b2)-(d2-a2)*(c1-b1), 2*e1-(a1+c1), 2*e2-(a2+c2), 2*f1-(b1+d1), 2*f2-(b2+d2), (m1-f1)*(f2-e2)-(m2-f2)*(f1-e1), (m1-a1)*(a2-b2)-(m2-a2)*(a1-b1), j^2-(b1-m1)^2-(b2-m2)^2, k^2-(a1-m1)^2-(a2-m2)^2, (j-k)>, {a1,a2,b1,b2,c1,c2,d1});
```

(0)

(5)



**TRUE ON PARTS**

# Gelernter's Thm. 2 with Maple: Decomposition

```
PP:=PrimeDecomposition(Hypo);  
PP := <j + k, a1 + b1 - 2 m1, a2 + b2 - 2 m2, -2 e1 + a1 + c1, [...]
```

Next, we check which of these components are non-degenerate (i.e. not including any px restriction involving the free variables of the construction):

```
for i from 1 to 2 do HilbertDimension(PP[i]),  
EliminationIdeal(PP[i], {a1,a2,b1,b2,c1,c2,d1}) od;  
7, <0>  
7, <0>
```

} Non-degenerate

```
for i from 3 to 8 do HilbertDimension(PP[i]),  
EliminationIdeal(PP[i], {a1,a2,b1,b2,c1,c2,d1}) od;  
6, <b1 - a1, c1 - a1, d1 - a1>  
6, <b1 - a1, c1 - a1, d1 - a1>  
6, <b1 - a1, c1 - a1, d1 - a1>  
6, <b1 - a1, c1 - a1, d1 - a1>  
6, <c1 - b1, d1 - a1>  
6, <c1 - b1, d1 - a1>
```

} Degenerate

```
for i from 9 to 12 do HilbertDimension(PP[i]),  
EliminationIdeal(PP[i], {a1,a2,b1,b2,c1,c2,d1}) od;  
7, <b1 - a1 + d1 - c1>  
7, <b1 - a1 + d1 - c1>  
7, <-a1 b2 + a1 c2 + a2 b1 - a2 c1 - b1 c2 + c1 b2>  
7, <-a1 b2 + a1 c2 + a2 b1 - a2 c1 - b1 c2 + c1 b2>
```

# Gelernter's Thm. 2 with Maple: Decomposition

```
for i from 1 to 2 do EliminationIdeal(PP[i]+<t*(j-k)-1>,{a1,  
a2,b1,b2,c1,c2,d1}) od;
```

<0>

<1>

Not generally true in PP[1]  
Generally true in PP[2]

```
for i from 1 to 2 do EliminationIdeal(PP[i]+<(j-k)>,{a1,a2,b1,  
b2,c1,c2,d1}) od;
```

<a1^2 - 2a1b1 + a2^2 - 2a2b2 + b1^2 + b2^2>

<0>

Generally false in PP[1]  
Not generally false in PP[2]

```
PP[1]:=<j + k, a1 + b1 - 2*m1, a2 + b2 - 2*m2, -2*e1 + a1 + c1, -2*e2 + a2 + c2,  
d1 - 2*f1 - a1 + 2*m1, 2*m2 - a2 - 2*f2 + d2, -e1*f2 + e1*m2 + e2*f1 - e2*m1  
- f1*m2 + f2*m1, -a1^2 + 2*a1*m1 - a2^2 + 2*a2*m2 + j^2 - m1^2 - m2^2>;
```

```
PP[2]:=<k - j, a1 + b1 - 2*m1, a2 + b2 - 2*m2, -2*e1 + a1 + c1, -2*e2 + a2 + c2,  
d1 - 2*f1 - a1 + 2*m1, 2*m2 - a2 - 2*f2 + d2, -e1*f2 + e1*m2 + e2*f1 - e2*m1  
- f1*m2 + f2*m1, -a1^2 + 2*a1*m1 - a2^2 + 2*a2*m2 + j^2 - m1^2 - m2^2>
```

# Gelernter's Thm. 2: MEP

Use  $j^2 = k^2$  instead  $j = k$  as thesis

gelernter 2.ggb

CAS

Graphics

12 Let A, B, C be arbitrary points.

13 Let f be the line B, C.

14 Let g be the line through A parallel to f.

15 Let D be a point on g.

16 Let E be the midpoint of A, C.

17 Let F be the midpoint of B, D.

18 Let h be the line F, E.

19 Let i be the line A, B.

20 Let M be the intersection of i and h.

21 Let j be the segment B, M.

22 Let k be the segment M, A.

23 Prove that  $j^2 \neq k^2$ .

24 The statement is true under some non-degeneracy conditions

25 We prove this by contradiction.

26 Let free point A be denoted by (v73,v74).

CAS

75  $s8: -v87*v76+v88*v75=0$   
 $\rightarrow s8: v75 v88 - v76 v87 = 0$

76  $s9: v87*v86-v88*v85-v87*v84+v85*v84+v88*v83-v86*v83=0$   
 $\rightarrow s9: -v83 v86 + v83 v88 + v84 v85 - v84 v87 - v85 v88$

77  $s10: v89^2-v88^2-v87^2+2*v88*v76-v76^2+2*v87*v75-v75^2=0$   
 $\rightarrow s10: -v75^2 - v76^2 - v87^2 - v88^2 + v89^2 + 2 v75 v87 +$

78  $s11: v90^2-v88^2-v87^2=0$   
 $\rightarrow s11: -v87^2 - v88^2 + v90^2 = 0$

79  $s12: -1-v91*v90^2+v91*v89^2=0$   
 $\rightarrow s12: v89^2 v91 - v90^2 v91 - 1 = 0$

80 Now we consider the following expression:

81  $s1*(-8*v86*v87^2*v91^2*v92^2*v75^4*v76*v81+4*v86*v87*v91$   
 $\rightarrow 1 = 0$

82 **Contradiction! This proves the original statement.**

83 The statement has a difficulty of degree 13.

**Remark:** high complexity, 13

**Pending work:** measure of the complexity of statements after computing and adding non-degeneracy conditions.

# Discussion

- **GeoGebra Discovery**: mathematical reliability and well performance even for complicated statements.
  - But does not output human-like explanations, just proof-certificates and estimation of difficulty, ShowProof (to be improved)
- **ChatGPT**: some times correct and readable proofs.
  - But, it makes mistakes and it is not stable.



## Close cooperation between

- **Chatbots**: readable proofs
- **GeoGebra Discovery/Maple**:
  - **Automatically** control the validity of the steps of the proof described by the bot, and
  - **Human** use, from the proof certificate answer, to verify by other computational tools the correction of the decisions of ChatGPT

# Conclusions

The mixing of the natural language abilities of a chatbot, with the recently extended features of GeoGebra Discovery and the power of Maple, could provide

- ❑ a rigorous checking of the mathematical truth of a given statement, drawn in the dynamic geometry window of this program,
- ❑ the availability of the corresponding algebraic translation, for analyzing and obtaining, through a CAS, further insight about its geometry, that could be quite relevant in special cases (e.g. true on parts),
- ❑ the production of proof-certificates, and
- ❑ the estimation of the difficulty of geometric theorems.

**A final remark:** the final goal is not to prove, but to learn more

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